

## Related Equations

### Flux

Flux is defined as the permeate flow divided by the total membrane surface area, as shown in the formula below, and is often presented in units of liters per square meters of membrane surface area per hour (gallons per day per square foot). Because the flux is greatly affected by the water temperature, the flux is often normalized to a standard temperature of 25° C (77° F) to account for fluctuations in water viscosity:

$$J = \frac{Q_p}{A_{System}}$$

Where

$J$  = flux (L/m<sup>2</sup>·h [gpd/sq ft]),

$Q_p$  = permeate flow (L/h [gpd]), and

$A_{System}$  = surface area of the membrane system (m<sup>2</sup> [sq ft]).

Flux can also be calculated as follows:

$$J = A_p(P_T - P_O)$$

Where

$A_p$  = the membrane permeability coefficient, which is the reciprocal of resistance to flow (also referred to by the letter  $K$  in other references, or mass-transfer coefficient [MTC] as defined below) (dimensionless),

$P_T$  = the transmembrane pressure (TMP) (kPa [psi]), and  
 $P_O$  = the osmotic pressure of the feed solution (kPa [psi]).

The above equation clearly shows that for water (feed solution) to flow through the membrane, the TMP must be greater than the osmotic pressure of the solution to provide a positive driving force.

The effect of temperature on flux can be evaluated as follows:

$$J_T = J_{25} \times 1.03^{(T - 25)}$$

Where

$J_T$  = the flux at temperature  $T$  (°C) and  
 $J_{25}$  = the flux at temperature 25°C (77°F).

The 25° C (77° F) reference temperature is used in nanofiltration (NF)/reverse osmosis (RO). For MF/UF, 20°C (68°F) is used for correcting the permeate flux to a reference temperature. The water flux typically increases by 3% for each degree temperature increase.

Therefore, to be able to evaluate changes in system in performance over time, all data must be “normalized” to a constant temperature. Figure AI.1 below shows an example of a normalized plot of MTCs for a pressurized MF system treating unchlorinated secondary effluent. For low-pressure membrane processes, common practice is to normalize flux data to 20° C (68° F) using one of the following equations. The expressions within brackets are correlations of viscosity with temperature:

$$J_{20} = J_T \cdot \text{TCF} \rightarrow \text{TCF} = \frac{\mu_T}{\mu_{20}}$$

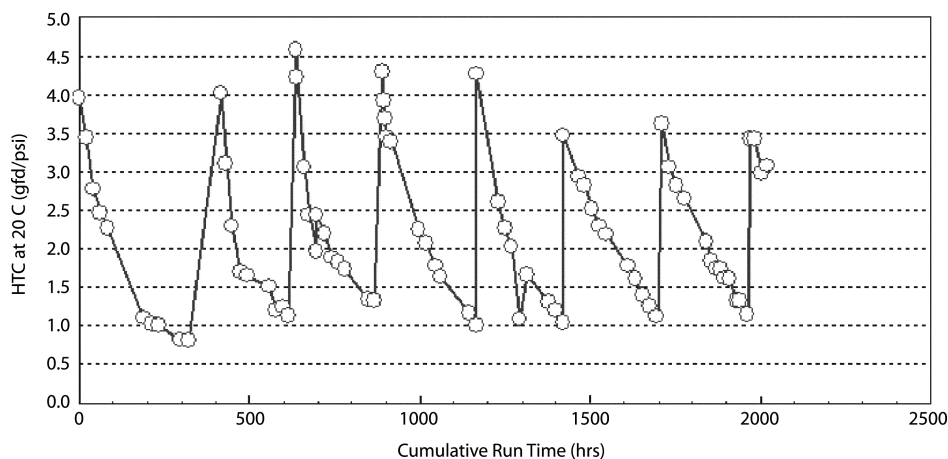
$$J_{20} = J_T \cdot \left[ 1.784 - (0.0575 \cdot T) + (0.001 \cdot T^2) - (10^{-5} \cdot T^3) \right]$$

or

$$J_{20} = J_T \cdot e^{[-0.032 \cdot (T - 20)]}$$

Where

$J_{20}$  = normalized flux at 20°C (L/m<sup>2</sup>·h [gpd/sq ft]),  
 $J_T$  = actual flux at temperature  $T$  (L/m<sup>2</sup>·h [gpd/sq ft]),



**FIGURE AI.1—EXAMPLE NORMALIZED MASS-TRANSFER COEFFICIENT PLOT FOR A PRESSURE MF SYSTEM ( $\text{gpd/sq ft/psi} \times 0.246 = \frac{\text{L/m}^2 \cdot \text{h}}{\text{kPa}}$ ) (COURTESY OF WATER ENVIRONMENT RESEARCH FOUNDATION).**

- $T$  = water temperature ( $^{\circ}\text{C}$ ),
- TCF = temperature correction factor,
- $\mu_{20}$  = viscosity of water at  $20^{\circ}\text{C}$  ( $\text{cP} [\text{Pa}\cdot\text{s}] = 1.0$ , and
- $\mu_T$  = viscosity of water at temperature  $T$ ,  $^{\circ}\text{C}$  ( $\text{cP} [\text{Pa}\cdot\text{s}]$ ).

## Total Surface Area

The total surface area represents the total membrane surface area available for treatment in a membrane system. This may be calculated by multiplying the surface area contained within each element by the number of elements contained in the membrane system, as shown by:

$$A_{\text{System}} = A_{\text{Element}} \times N_{\text{Element}}$$

Where

- $A_{\text{System}}$  = total surface area ( $\text{m}^2$  [sq ft]),
- $A_{\text{Element}}$  = surface area of each element ( $\text{m}^2$  [sq ft]), and
- $N_{\text{Element}}$  = number of elements in membrane system.

## Transmembrane Pressure

The TMP is defined as the difference between the average feed/concentrate pressure and the permeate pressure, as shown below. It is effectively the driving force for flux. The TMP of the membrane system is an overall indication of the feed pressure requirement; it is used, with the flux, to assess membrane fouling.

For crossflow mode of operation:

$$\text{TMP} = \left( \frac{P_F + P_C}{2} \right) - P_P$$

Where

TMP = transmembrane pressure (kPa [psi]),

$P_F$  = feed pressure (kPa [psi]),

$P_C$  = concentrate pressure (kPa [psi]), and

$P_P$  = permeate pressure (kPa [psi]).

For the direct-feed mode of operation:

$$\text{TMP} = P_F - P_P$$

It should be noted that, with MF/UF, the feed pressure and concentrate pressure are equal.

## Net Driving Pressure

Net driving pressure (NDP) is the pressure available to drive the feed water through the membrane minus the permeate and osmotic backpressure:

$$\text{NDP} = P_F - \frac{1}{2}(P_F - P_C) - P_P - \Delta\pi$$

Where

$P_F$  = feed pressure (kPa [psi]),

$P_C$  = concentrate pressure (kPa [psi]),

$P_P$  = permeate or filtrate pressure (kPa [psi]), and

$\Delta\pi$  = change in osmotic pressure (kPa [psi]).

## Recovery

The percentage of feed that is converted to permeate is called the recovery (water or liquid) of the membrane system and is calculated by the formula below (see Figure AI.2). Recovery is critical in NF and RO, as not all of the liquid will go through the membrane surface. In MF and UF, the liquid stream applied to the membrane surface will go through. Generally, it is desired to operate at a high recovery, as it minimizes the waste stream. However, operating at elevated recoveries may result in increased fouling rates and cleaning frequencies. Manufacturers should be consulted before altering the operating recovery of the membrane system.

$$R = \frac{Q_P}{Q_F} \times 100\%$$

Where

$R$  = recovery (%),

$Q_P$  = permeate flow (L/s [gpm]), and

$Q_F$  = feed flow (L/s [gpm]).

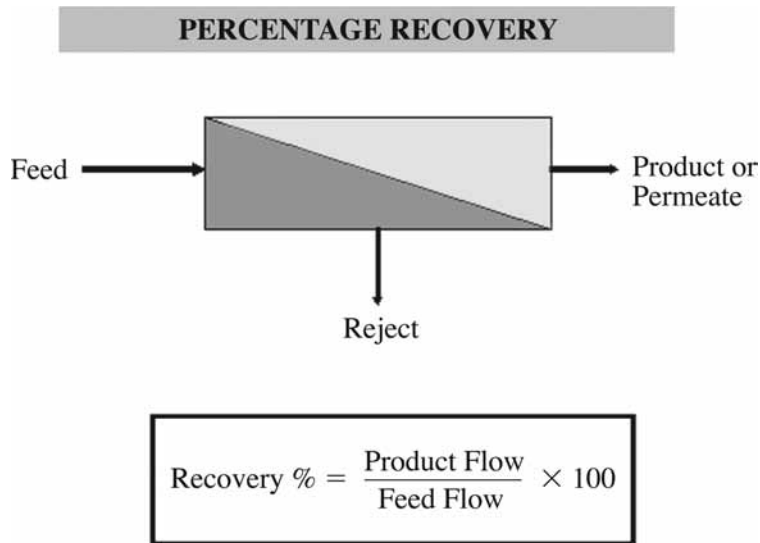


FIGURE AI.2—SCHEMATIC DEPICTING PERCENT RECOVERY.

## Contaminant Removal (or Rejection)

Contaminant removal is defined as the percentage of a contaminant removed from the feed stream by the membrane and may be calculated by the formula shown below. Contaminant removal may be calculated for any parameter of interest (turbidity, total suspended solids, total organic carbon, etc.); however, consistent units must be maintained throughout the calculation:

$$R_{\text{Cont}} = \frac{C_F - C_P}{C_F} \times 100\%$$

Where

$R_{\text{Cont}}$  = contaminant removal (%),

$C_F$  = feed contaminant concentration (e.g., mg/L), and

$C_P$  = permeate contaminant concentration (e.g., mg/L).

## Backwash Volume

Microfiltration (MF) and ultrafiltration (UF) backwash volumes can be estimated with the following equation:

$$V = \text{No. of backwashes per day} \times \text{Volume per backwash} \\ = \frac{1440 \text{BW}_V}{\text{BW}_{\text{int}}}$$

Where

$\text{BW}_V$  = volume of water used per backwash (L) and

$\text{BW}_{\text{int}}$  = backwash interval in minutes (minutes).

A mass balance around the membrane process can be used to estimate the concentration of feed water constituents in the backwash water according to the following equation:

$$C_{\text{BW}} = \frac{C_f}{1 - r}$$

Where

$C_f$  = concentration of constituent in the feed water and  
 $r$  = system recovery.

## Langelier Saturation Index

The most common method for determining the solubility of calcium carbonate ( $\text{CaCO}_3$ ) in water is the Langelier saturation index (LSI). Waters that are negative on this index indicate an undersaturated condition with respect to  $\text{CaCO}_3$  (tendency to dissolve  $\text{CaCO}_3$ ), whereas waters that are positive indicate an oversaturated condition (tendency to precipitate  $\text{CaCO}_3$ ). The LSI equation is as follows:

$$\text{LSI} = \text{pH} - \text{pH}_s$$

$$\text{pH}_s = (9.3 + A + B) - (C + D)$$

Where

$\text{pH}_s$  = saturation pH for  $\text{CaCO}_3$ ,

$$A = \frac{(\log_{10}[\text{Total dissolved solids}] - 1)}{10}$$

$$B = -13.12 \times \log_{10}(^{\circ}\text{C} + 273) + 34.55,$$

$$C = \log_{10}[\text{Ca}^{+2} \text{ as } \text{CaCO}_3] - 0.4, \text{ and}$$

$$D = \log_{10}[\text{Alkalinity as } \text{CaCO}_3].$$

## Stiff and Davis Scaling Index

Other equations used for determining the solubility of calcium carbonate include the Stiff and Davis scaling index (SDSI). The SDSI equation is as follows:

$$\text{SDSI} = \text{pH} - p_{\text{Ca}} - p_{\text{Alk}} - K$$

Where

$p_{Ca}$  = negative logarithm of the calcium molarity, as in LSI;

$p_{Alk}$  = negative logarithm of the alkalinity in equivalents per liter, as in LSI; and

$K$  = different empirical constant from LSI to account for temperature and ionic strength.

## Silt Density Index

The tendency for the water feed to foul a membrane can be evaluated with a filterability test called the silt density index (SDI). The SDI is primarily applicable in NF/RO. The test is described in the ASTM standard no. D4185. The test is very simple and consists of a 0.45- $\mu\text{m}$  ( $1.8 \times 10^{-5}$ -in.) cellulose acetate membrane in a dead-end filtration cell. The test is conducted for 15 minutes, labeled the total test time,  $T_t$  or  $T_{15}$ . The time, in minutes, needed to collect the initial 500 mL (30.5 cu in.) of filtrate is recorded as  $T_i$ . The time needed to collect another 500 mL (30.5 cu in.) of filtrate after the filter has been online for 15 minutes is recorded as  $T_f$ . Standard conditions for the SDI determination call for a 47-mm-diam (1.85-in.-diam) filter and an applied pressure of 206.8 kPa (30 psi or 2 bar) and a total test time of 15 minutes. The SDI is calculated according to

$$\text{SDI} = \frac{100 \left( 1 - \frac{T_i}{T_f} \right)}{T_t}$$

Where

$T_i$  = time in minutes needed to collect the initial 500 mL (30.5 cu in.),

$T_f$  = time needed to collect 500 mL (30.5 cu in.) after being online for 15 minutes, and

$T_t$  = total time of the test, 15 minutes.

For successful operation of hollow-fiber and spiral-wound RO membranes, a SDI of 2 to 3 is desirable in desalting membranes, with 3 to 5 as an upper limit. For best performance, membrane manufacturers will recommend the SDI limit. Exceeding the SDI limit will require pretreatment of the

feed to the membrane and may require some operational changes, such as reduced flux rate. With lower SDI, pretreatment may be advisable, as it reduces the cleaning cycle.

## Solubility Product

The solubility product for a given salt can be found using the following equation:

$$K_{sp} = [\text{Cation}]^{\#} \times [\text{Anion}]^{\#}$$

Where

$K_{sp}$  = a value for the solubility product,

[Cation] = the cation concentration,

[Anion] = the anion concentration, and

# = the quantity of the particular ion present within the salt molecule.

